13 Statistics

Fastrack Revision

▶ **Arithmetic Mean or Mean:** The arithmetic mean or mean is the value obtained by dividing the sum of values of the items in a series by the number of items.

1. Mean of Ungrouped Data:

(i) Direct Method: The mean of n observations x_1, x_2, x_3,..., x_n is

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}, \text{ i.e., } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

(ii) Frequency Distribution: Let f_1, f_2, f_3,..., f_n be the corresponding frequencies of x_1, x_2, x_3,..., x_n. Then

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

(iii) Assumed Mean Method (Short Cut Method): Let 'A' be the assumed mean from given data, then

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}, \text{ where deviation } d_i = x_i - A$$

2. Mean of Grouped Data: The class mark of the class interval $a_i - b_i$, is the mid value of a_i and b_i, *i.e.*, $(x_i) = \frac{a_i + b_i}{2}$.

(i) Direct Method:

$$\text{Mean } (\bar{x}) = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots.. + f_nx_n}{f_1 + f_2 + f_3 + \ldots. + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} \frac{1}{2} f_i(a_i + b_i)}{\sum_{i=1}^{n} f_i}$$

(ii) Assumed Mean Method (Short Cut Method): Let 'A' be assumed mean occurring in the middle of the mid values of the class interval, then

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i d_i}{\Sigma f_i}, \text{ where deviation } d_i = x_i - A$$

(iii) Step-deviation Method:

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h, \text{ where } u_i = \frac{x_i - A}{h} \text{ and } h \text{ is}$$

width of class interval.

Knowledge BOOSTER

1. *The algebraic sum of the deviation of variables from their mean is always zero* (0), *i.e.*, $\sum_{i=1}^{n} f_i(x_i - \bar{x}) = 0$.
2. *Combined Mean* $(\bar{x}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$
3. *If each observation is increased by* 5 (*or decreased by* 5), *then the resultant mean is increased by* 5 (*or decreased by* 5).
4. *There is no need to convert discontinuous data into continuous data, while finding the mean of the data.*

▶ **Cumulative Frequency:** It is defined as the sum of all frequencies up to the given class.

▶ **Mode:** The observation which is repeated (occur) maximum number of times in the data or has the highest frequency is called mode.

1. When the Data is Ungrouped: The observation which occurs maximum number of times in the given data or has the highest frequency would be the mode of given data.

2. When the Data is Grouped:

$$\text{Mode} = l + \left(\frac{f_m - f_p}{2f_m - f_p - f_s}\right) \times h$$

where, l = Lower limit of the modal class,

f_m = Frequency of the modal class,

f_p = Frequency of the class preceding the modal class,

f_s = Frequency of the class succeeding the modal class,

h = Size of the class interval.

▶ **Median:** The median of a distribution is the value of the middle variable when the variables are arranged in ascending or descending order.

1. Median of Ungrouped Data:

(i) (a) Arrange the data in ascending or descending order and then find the number of terms n.

(b) If n is odd, then median = $\left(\frac{n+1}{2}\right)$ th observation.

(c) If n is even, then median.

$$= \frac{1}{2}\left\{\begin{matrix}\left(\frac{n}{2}\right)\text{th observation} \\ +\left(\frac{n}{2}+1\right)\text{th observation}\end{matrix}\right\}$$

(ii) In Frequency Distribution:

(a) First find $\frac{N}{2}$, where $N = \Sigma f$

(b) Find the cumulative frequency of each value of the variable and take the value which is equal to or just greater than $N/2$

(c) This value of the variable is the median.

2. Median of Grouped Data:

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - cf}{f} \times h\right\}$$

where, l = Lower limit of median class,

f = Frequency of the median class,

cf = Cumulative frequency of the class preceding the median class,

h = Size of the class interval,

N = Sum of the frequencies.

▶ **Empirical Formula:** Mode = 3 (Median) – 2 (Mean)

Practice Exercise

Multiple Choice Questions ↘

Q 1. Which of the following is a measure of central tendency?

a. Class limit b. Lower limit

c. Cumulative frequency d. Median

Q 2. While computing mean of grouped data, we assume that the frequencies are:

a. centered at the class marks of the class

b. centered at lower limits of the class

c. centered at upper limits of the class

d. None of the above

Q 3. If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean, then $\Sigma f_i(x_i - \overline{x})$ is equal to: *[NCERT EXEMPLAR]*

a. 0 b. –1 c. 1 d. 2

Q 4. In the formula, $\overline{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$ for finding the mean of grouped data, d_i's are deviations from A of: *[NCERT EXEMPLAR]*

a. lower limits of the classes

b. upper limits of the classes

c. mid-points of the classes

d. frequencies of the class marks

Q 5. Construction of a cumulative frequency table is useful in determining the: *[NCERT EXEMPLAR]*

a. mean b. mode

c. median d. All of these

Q 6. The range of the following data is:

Diameter (in mm)	30-40	40-50	50-60	60-70	70-80
No. of screws	10	15	18	12	7

a. 10 b. 30 c. 50 d. 70

Q 7. In the following distribution table, the preceding frequency value of modal class is:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	2	5	18	14	8

a. 8 b. 5

c. 18 d. 14

Q 8. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The lower limit of modal class: *[CBSE SQP 2023-24]*

a. 15 b. 25

c. 30 d. 35

Q 9. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of the class 30-40 is: *[NCERT EXEMPLAR]*

a. 3 b. 4

c. 48 d. 51

Q 10. Consider the following frequency distribution:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is: *[NCERT EXEMPLAR]*

a. 17 b. 17.5

c. 18 d. 18.5

Q 11. If median of the following data is 345,

Daily wages (in ₹)	Number of workers
320-330	10
330-340	15
340-350	8
350-360	7
360-370	8

Then, find the median class.

a. 330-340 b. 350-360

c. 340-350 d. Can't be determined

Q 12. **Consider the following frequency distribution:**

Class	0-6	6-12	12-18	18-24	24-30
Frequency	12	10	15	8	11

The median class is: *[CBSE SQP 2023-24]*

a. 6-12 b. 12-18
c. 18-24 d. 24-30

Q 13. **For the following distribution:**

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of lower limits of the median class and modal class is: *[NCERT EXEMPLAR, CBSE SQP 2022-23, CBSE 2023]*

a. 15 b. 25 c. 30 d. 35

Q 14. **Consider the data:**

Class	Frequency
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	7
185-205	4

The difference of the upper limit of the median class and the lower limit of the modal class is: *[NCERT EXEMPLAR]*

a. 0 b. 19 c. 20 d. 38

Q 15. **Median and Mode of a distribution are 25 and 21 respectively. Mean of the data using empirical relationship is:** *[CBSE 2023]*

a. 27 b. 29 c. 18 d. $\frac{29}{3}$

Assertion & Reason Type Questions

Directions (Q. Nos. 16-20): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 16. **Assertion (A): If $\Sigma f_i = 20$, $\Sigma f_i x_i = 3\lambda + 20$ and mean of the distribution is 4, then the value of λ is 20.**

Reason (R): If there are $x_1, x_2, \ldots\ldots, x_n$ observations where corresponding frequencies are $f_1, f_2, \ldots\ldots f_n$, then mean is determined by the formula,

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

Q 17. **Assertion (A): The mode of the following frequency distribution is 52.25 kg.**

Weight (in kg)	Frequency
40-45	20
45-50	30
50-55	35
55-60	20
60-65	10

Reason (R): A modal class is a class which has highest frequency.

Q 18. **Assertion (A): The median of the frequency distribution is 68.75.**

Marks obtained (Class Interval)	Number of students (Frequency)	Cumulative frequency
30-40	5	5
40-50	10	15
50-60	3	18
60-70	8	26
70-80	14	40
80-90	10	50

Reason (R): The cumulative frequency of median is just next to the median class.

Q 19. **Assertion (A): The following table gives the marks scored by students in an examination:**

Marks	Number of students
0-5	3
5-10	7
10-15	15
15-20	24
20-25	16
25-30	8

The succeeding frequency of modal class is 16.
Reason (R): The sum of frequency of modal class and its preceding frequency is 40.

Q 20. **Assertion (A): If the median and mode of a frequency distribution are 150 and 154 respectively, then its mean is 148.**
Reason (R): The relation between mean, mode and median is:

Mode = 3 (Median) – 2 (Mean)

Fill in the Blanks Type Questions

Q 21. **The mean of first 10 prime numbers is .....**

Q 22. **If the mean of 18 observations is 25 and each observation is decreased by 5, the new mean will be .....**

Q 23. **In the formula $\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$ for finding the mean of grouped frequency distribution, $u_i =$ .....** *[NCERT EXEMPLAR]*

Q 24. ..... is the value of the middle variable when the variables are arranged in ascending or descending order.

Q 25. If the values of mean and mode are respectively 30 and 15, then median is .....

True/False Type Questions

Q 26. The algebraic sum of the deviation of variables from their mean is always zero.

Q 27. The value of x, if the mean of 5 observations x, $x+2$, $x+4$, $x+6$ and $x+8$ is 11, will be 7.

Q 28. In the frequency distribution, if $\Sigma f_i x_i = 750$ and $\Sigma f_i = 25$, the mean of the distribution is 25.

Q 29. If the values of mean and median are 53.6 and 55.81, then the value of mode is 60.23.

Q 30. The preceding frequency is just after the modal class.

Solutions

1. (d) Median

2. (a) centered at the class marks of the class.

3. (a) Given, $\bar{x}$ is a mean.

Therefore, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad \bar{x}\,\Sigma f_i = \Sigma f_i x_i$

Now, $\Sigma f_i (x_i - \bar{x}) = \Sigma f_i x_i - \bar{x}\,\Sigma f_i = \Sigma f_i x_i - \Sigma f_i x_i = 0$

4. (c) mid-points of the classes

5. (c) median

6. (c) Here, upper class boundary of the highest interval $= 80 = u$.

and lower class boundary of the lowest interval $= 30 = l$

$\therefore$ Range of grouped data $= u - l = 80 - 30 = 50$

7. (b) In the given distribution table, the highest frequency is 18. Therefore its modal class is 10-15.

TiP

The preceding frequency value of modal class is just before the modal class.

Hence, the preceding frequency value of modal class is 5.

8. (a) From the given distribution, highest frequency is 20, which lies in the class interval 15-20. This class is said to be modal class. So, lower limit of the modal class is 15.

9. (a) The frequency of the class 30-40 = Number of students having more than or equal to 30 marks – Number of students having more than or equal to 40 marks

$= 51 - 48 = 3$

10. (b)

TiP

Given frequency table is not continuous, so make continuous by subtracting −0.5 in lower limits and adding 0.5 in the upper limits.

The cumulative frequency table is given below:

Class	Frequency	*cf*
−0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $N = 57 \Rightarrow \frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the cumulative frequency 38, whose median class is 11.5-17.5. The upper limit of median class is 17.5.

11. (c) Given median is 345, which lies in the interval 340-350.

12. (b) The cumulative frequency of given data is shown below:

Class	Frequency (*f*)	Cumulative frequency (*cf*)
0-6	12	12
6-12	10	22
12-18	15	37
18-24	8	45
24-30	11	56

Here, $\frac{N}{2} = \frac{56}{2} = 28$, since 28 lies in the cumulative frequency 37. So, median class is 12-18.

13. (b) The cumulative frequency is shown below:

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Here, $N = 66 \Rightarrow \frac{N}{2} = \frac{66}{2} = 33$, which lies in the cumulative frequency 37. So, median class is 10-15.

In the given table, the highest frequency is 20. So, modal class is 15-20.

Here, lower limits of median class and modal class are 10 and 15 respectively.

Therefore, the sum of lower limits of median class and modal class is 10 + 15, *i.e.*, 25.

14. (c) The cumulative frequency table is shown below:

Class	Frequency	Cumulative frequency
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $N = 67 \Rightarrow \frac{N}{2} = \frac{67}{2} = 33.5$, which lies in the cumulative frequency 42. So, median class is 125-145.

In the given table, the highest frequency is 20.

So, modal class is 125-145.

Here, upper limits of median class and lower limit of modal class are 145 and 125 respectively.

Therefore, the difference of upper limits of median class and lower limit of modal class is $145 - 125 = 20$.

15. (a) Given,

$$\text{Median} = 25 \text{ and Mode} = 21$$

$$\because \quad \text{Mode} = 3\,(\text{Median}) - 2\,(\text{Mean})$$

$$2(\text{Mean}) = 3\,(\text{Median}) - \text{Mode}$$

$$= 3 \times 25 - 21$$

$$= 75 - 21 = 54$$

$$\Rightarrow \quad \text{Mean} = \frac{54}{2} = 27$$

16. (a) **Assertion (A):** Given, $\Sigma f_i = 20$, $\Sigma f_i x_i = 3\lambda + 20$ and mean of data is $\bar{x} = 4$

Then $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$$\Rightarrow \quad 4 = \frac{3\lambda + 20}{20}$$

$$\Rightarrow \quad 3\lambda + 20 = 80$$

$$\Rightarrow \quad 3\lambda = 60 \Rightarrow \lambda = 20.$$

So, Assertion (A) is true.

Reason (R): It is true to say that in frequency distribution, the mean is determined by $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

17. (d) **Assertion (A):** In the frequency distribution, the highest frequency is 35, which lies in the class interval 50-55.

So, modal class is 50-55.

Here, $l = 50$, $f_m = 35$, $f_p = 30$ and $f_s = 20$, $h = 5$

$$\because \quad \text{Mode} = l + \left(\frac{f_m - f_p}{2f_m - f_p - f_s}\right) \times h$$

$$= 50 + \left(\frac{35 - 30}{2 \times 35 - 30 - 20}\right) \times 5$$

$$= 50 + \frac{25}{70 - 50} = 50 + \frac{25}{20}$$

$$= 50 + 1.25 = 51.25 \text{ kg.}$$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason is true.

18. (c) **Assertion (A):** Given, the sum of frequencies is $N = 50$.

$\therefore \quad \frac{N}{2} = \frac{50}{2} = 25$, which lies in the cumulative frequency 26.

So, median class is 60-70.

Here, $l = 60$, $f = 8$, $cf = 18$ and $h = 10$

$$\because \quad \text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 60 + \frac{25 - 18}{8} \times 10$$

$$= 60 + \frac{70}{8} = 60 + 8.75 = 68.75$$

So, Assertion (A) is true.

Reason (R): Given Reason (R) is false, because cumulative frequency is just before the median class.

Hence, Assertion (A) is true and Reason (R) is false.

19. (c) **Assertion (A):** In a given table, the highest frequency is 24, whose modal class is 15-20.

The succeeding frequency of modal class is 16.

So, Assertion (A) is true.

Reason (R): In the given table, frequency of modal class is 24 and preceding frequency is 15.

$\therefore$ The sum of modal class frequency and preceding frequency is $24 + 15$, *i.e.*, 39.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

20. (a) **Assertion (A):** Given that,

$$\text{median} = 150 \text{ and mode} = 154$$

By using empirical relation,

$$\text{mode} = 3\,(\text{median}) - 2\,(\text{mean})$$

$$\Rightarrow \quad 154 = 3 \times 150 - 2\,(\text{mean})$$

$$\Rightarrow \quad 2\,(\text{Mean}) = 450 - 154$$

$$\Rightarrow \quad \text{Mean} = \frac{296}{2} = 148.$$

So, Assertion (A) is true.

Reason (R): It is true to say that the relation between mean, mode and median is

$$\text{Mode} = 3\,(\text{Median}) - 2\,(\text{Mean})$$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21.

TR!CK

First 10 *prime numbers are* 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Mean of first 10 prime numbers

$$= \frac{\text{Sum of all the observations}}{\text{Total number of observations}}$$

$$= \frac{2+3+5+7+11+13+17+19+23+29}{10}$$

$$= \frac{129}{10} = 12.9$$

22. Let 18 observations be $x_1, x_2, \ldots\ldots x_{18}$.

Then, mean of 18 observations is

$$25 = \frac{\sum_{i=1}^{18} x_i}{18} \quad \ldots(1)$$

When 5 is decreased in each observation, then new mean

$$\bar{x} = \frac{\sum_{i=1}^{18}(x_i - 5)}{18}$$

$$= \frac{\sum_{i=1}^{18} x_i - \sum_{i=1}^{18} 5}{18}$$

$$= \frac{\sum_{i=1}^{18} x_i}{18} - \frac{(5+5+\ldots\ldots 18 \text{ times})}{18}$$

$$= 25 - \frac{18 \times 5}{18} \quad \text{[from eq. (1)]}$$

$$= 25 - 5 = 20$$

23. $\frac{x_i - a}{h}$

24. Median

25. Given, mean = 30 and mode = 15

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

$$15 = 3\,(\text{Median}) - 2 \times 30$$

$$\Rightarrow \quad 3\,(\text{Median}) = 15 + 60$$

$$\Rightarrow \quad \text{Median} = \frac{75}{3} = 25$$

26. True

27. $$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

$$\Rightarrow \quad 11 = \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5}$$

$$\Rightarrow \quad 55 = 5x + 20$$

$$5x = 35$$

$$\Rightarrow \quad x = 7.$$

Hence, given statement is true.

28. Given, $\Sigma f_i x_i = 750$ and $\Sigma f_i = 25$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{750}{25} = 30$$

Hence, given statement is false.

29. Given, mean = 53.6 and median = 55.81

$$\because \quad \text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

$$= 3 \times 55.81 - 2 \times 53.6$$

$$= 167.43 - 107.2$$

$$= 60.23$$

Hence, given statement is true.

30. False, because preceding frequency is just before the modal class.

Case Study Based Questions

Case Study 1

An agency has decided to install customised playground equipments at various colony parks. For that they decided to study the age-group of children playing in a park of the particular colony. The classification of children according to their ages, playing in a park is shown in the following table:

Age group of children (in years)	Number of children
6-8	43
8-10	58
10-12	70
12-14	42
14-16	27

Based on the above information, solve the following questions:

Q 1. The maximum number of children are of the age-group:

a. 12-14 b. 10-12
c. 14-16 d. 8-10

Q 2. The lower limit of the modal class is:

a. 10 b. 12 c. 14 d. 8

Q 3. Frequency of the class succeeding the modal class is:

a. 58 b. 70 c. 42 d. 27

Q 4. The mode of the ages of children playing in the park is:

a. 9 yr b. 8 yr
c. 11.5 yr d. 10.6 yr

Q 5. If mean and mode of the ages of children playing in the park are same, then median will be equal to:

a. Mean b. Mode

c. Both a. and b. d. Neither a. nor b.

Solutions

1. (b) Since, the highest frequency is 70, therefore the maximum number of children are of the age-group 10-12. Which is also called modal class.

So, option (b) is correct.

2. (a) Since, the modal class is 10-12

$\therefore$ Lower limit of modal class (l) = 10

So, option (a) is correct.

3. (c) From the table, the frequency of the modal class is 70.

Thus, the frequency of the class succeeding the modal class is 42.

So, option (c) is correct.

4. (d) Here, $l = 10$, $f_1 = 70$, $f_0 = 58$, $f_2 = 42$ and $h = 2$

$$\therefore \quad \text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 10 + \left[\frac{70 - 58}{140 - 58 - 42}\right] \times 2$$

$$= 10 + \frac{12}{40} \times 2 = 10 + \frac{24}{40} = 10.6 \text{ yr}$$

So, option (d) is correct.

5. (c) Given that, Mean = Mode

$\therefore$ By Empirical relation, we have

Mode = 3 Median – 2 Mean

$\Rightarrow$ Mode = 3 Median – 2 Mode

$\Rightarrow$ 3 Mode = 3 Median

$\Rightarrow$ Median = Mode = Mean

So, option (c) is correct.

Case Study 2

A petrol pump owner wants to analyse the daily need of diesel at the pump. For this he collected the data of vehicles visited in 1 h. The following frequency distribution table shows the classification of the number of vehicles and quantity of diesel filled in them:

Diesel Filled (in Litres)	3-5	5-7	7-9	9-11	11-13
Number of vehicles	5	10	10	7	8

Based on the above information, solve the following questions:

Q 1. Which of the following is correct?

a. If x_i and f_i are sufficiently small, then direct method is appropriate choice for calculating mean.

b. If x_i and f_i are sufficiently large, then direct method is appropriate choice for calculating mean.

c. If x_i and f_i are sufficiently small, then assumed mean method is appropriate choice for calculating mean.

d. None of the above

Q 2. Average diesel required for a vehicle is:

a. 8.15 L b. 6 L c. 7 L d. 5.5 L

Q 3. If approximately 2000 vehicles comes daily at the petrol pump, then how much litres of diesel the pump should have?

a. 16200 L b. 16300 L

c. 10600 L d. 15000 L

Q 4. The sum of upper and lower limit of median class is:

a. 22 b. 10

c. 16 d. None of these

Q 5. If the median of given data is 8 L, then mode will be equal to:

a. 7.5 L b. 7.7 L c. 5.7 L d. 8 L

Solutions

1. (a) If f_i and x_i are very small, then direct method is appropriate method for calculating mean.

So, option (a) is correct.

2. (a) The frequency distribution table from the given data can be drawn as:

Class	Class mark (x_i)	Frequency (f_i)	$f_i x_i$	cf
3-5	4	5	20	5
5-7	6	10	60	15
7-9	8	10	80	25
9-11	10	7	70	32
11-13	12	8	96	40
Total		**40**	**326**	

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{326}{40} = 8.15 \text{ L}$$

So, option (a) is correct.

3. (b) If 2000 vehicles comes daily and average quantity of diesel required for a vehicle is 8.15 L, then total quantity of diesel required = 2000×8.15

= 16300 L

So, option (b) is correct.

4. (c) Here, $N = 40$ and $\frac{N}{2} = 20$

cf for the distribution are 5, 15, 25, 32, 40

Now, cf just greater than 20 is 25 which is corresponding to the class interval 7-9.

So median class is 7-9.

$\therefore$ Required sum of upper limit and lower limit

$= 7 + 9 = 16$

So, option (c) is correct.

5. (b) We know, Mode = 3 Median – 2 Mean
$= 3(8) - 2(8.15) = 24 - 16.3 = 7.7$L
So, option (b) is correct.

Case Study 3

A group of 71 people visited to a museum on a certain day. The following table shows their ages:

Age (in years)	Number of persons
Less than 10	3
Less than 20	10
Less than 30	22
Less than 40	40
Less than 50	54
Less than 60	71

Based on the above information, solve the following questions:

Q 1. If true class limits have been decided by making the classes of interval 10, then find the first class interval.

Q 2. Find the cumulative frequency table.

Q 3. Find the frequency of class preceding the median class.

Or

If the price of a ticket for the age group 30-40 is ₹ 30, then find the total amount spent by this age group.

Solutions

1. The age of any person is a positive number, so the first class interval must be 0-10.

2. Cumulative frequency table:

Age (in years)	Class interval (x_i)	Frequencies (f_i)	Cumulative frequency (cf)
Less than 10	0-10	3	3
Less than 20	10-20	$10 - 3 = 7$	10
Less than 30	20-30	$22 - 10 = 12$	22
Less than 40	30-40	$40 - 22 = 18$	40
Less than 50	40-50	$54 - 40 = 14$	54
Less than 60	50-60	$71 - 54 = 17$	71

3. From the table, $N = 71$, therefore $\frac{N}{2} = 35.5$

Now, the class interval whose cumulative frequency is just greater than 35.5 is 30-40.

$\therefore$ Median class = 30-40

So, the frequency of class preceding the median class is 2.

Or

Number of persons, whose age lying in 30-40 is 18.

$\therefore$ Total amount spent by people of this age group
$= ₹ (30 \times 18) = ₹ 540$

Case Study 4

Electric buses are becoming popular nowadays. These buses have the electricity stored in a battery. Electric buses could have a range of approximately 280 km with just one charge. Electric buses are superior to diesel buses as they reduce brake wear and also reduce pollution. Transport department of a city wants to buy some electric buses for the city. So, the department wants to know the distance travelled by existing public transport buses in a day.

(CBSE 2022 Term-2)

The following data shows the distance travelled by 50 existing public transport buses in a day:

Daily distance travelled (in km)	Number of buses
100-120	12
120-140	14
140-160	8
160-180	6
180-200	10

Based on the above information, solve the following questions:

Q 1. Write the relation between mean, median and mode.

Q 2. Find the modal class of the given distribution.

Q 3. Find the 'median' distance travelled by a bus.

OR

Find the 'mean (average)' distance travelled by a bus.

Solutions

1. The relation between mean, median and mode is:
Mode = 3 (median) – 2 (Mean)
Which is also called empirical formula.

2. From the given distribution, highest frequency is 14, which lies in the class interval 120-140. This class is said to be modal class.

3. The cumulative frequency table is given below:

Daily distance travelled (in km)	Class mark (x_i)	Number of buses (f_i)	Cumulative frequency (cf)	Deviation $u_i = \frac{x_i - A}{h}$	$F_i \times u_i$
100-120	110	12	12	–2	–24
120-140	130	14	26	–1	–14
140-160	150	8	34	0	0
160-180	170	6	40	1	6
180-200	190	10	50	2	20
		$\Sigma f = 50$			$\Sigma f_i u_i = -12$

Here, $N = 50$

$\therefore \frac{N}{2} = \frac{50}{2} = 25$, which lies in the cumulative.

frequency 26, whose corresponding class is in (120-140).

Thus (120-140) is a median class.

Here, $l = 120$, $cf = 12$, $f = 14$ and $h = 20$

$$\therefore \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 120 + \frac{(25-12)}{14} \times 20 = 120 + \frac{260}{14}$$

$$= 120 + 18.57 = 138.57$$

Hence the 'median' distance travelled by a bus is 138.57 km.

OR

$$\text{Now, Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f} \times h$$

(by step deviation method)

Here assumed mean (A) = 150.

$$\therefore \quad \text{Mean} = 150 + \frac{(-12)}{50} \times 20$$

$$= 150 - \frac{24}{5}$$

$$= 150 - 4.8 = 145.2$$

Hence, 'mean (average)' distance travelled by a bus is 145.2 km.

Very Short Answer Type Questions

Q 1. Find the class marks of the classes 10-25 and 35-55. *[CBSE 2020]*

Q 2. If $\sum f_i = 15$, $\sum f_i x_i = 3p + 36$ and mean of the distribution is 3, then find the value of p.

Q 3. The following table shows the gain in weight by 25 children in a year.

Gain in weight (x_i) (in kg)	1.5	2	2.4	3	3.2	3.4
Number of children (f_i)	4	5	8	5	2	1

Find the mean of gain in weight.

Q 4. If the mode of the data 3, 5, 8, 9, 8, 12, 7, 12 and x is 8, find the value of x. *[CBSE 2015]*

Q 5. Find the succeeding value of the modal class in the following data:

Marks	50-60	60-70	70-80	80-90	90-100
Number of students	4	8	14	19	5

Q 6. Find the median of 3, 9, 6, 12, 19 and 11.

Q 7. The following table gives production yield in kg per hectare of wheat of 100 farms of a village:

Production yield (kg/hectare)	Number of farms
40-45	4
45-50	6
50-55	16
55-60	20
60-65	30
65-70	24

Change the distribution to a 'more than type' distribution.

Q 8. Find the sum of the frequencies after the median class:

Height (in cm)	Number of students
140-150	5
150-160	6
160-170	10
170-180	4
180-190	7

Q 9. Find the median of the data, using an empirical relation when it is given that mode = 12.4 and mean = 10.5. *[CBSE 2016]*

Short Answer Type-I Questions

Q 1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

[NCERT EXERCISE]

Number of plants	Number of houses
0-2	1
2-4	2
4-6	1
6-8	5
8-10	6
10-12	2
12-14	3

Q 2. If the mean of the following data is 12, then find the value of p. *[CBSE 2017]*

x_i	4	8	p	16	20
f_i	5	3	12	5	4

Q 3. Find the mean of the following distribution by assumed mean method:

Class intervals	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

Q 4. The following is the distribution of weights (in kg) of 40 persons: *[CBSE 2017]*

Weight (in kg)	Number of persons
40–45	4
45–50	4
50–55	13
55–60	5
60–65	6
65–70	5
70–75	2
75–80	1

Construct a cumulative frequency distribution (of the less than type) table for the above data.

Q 5. If mode of the following frequency distribution is 55, then find the value of x: *[CBSE 2022 Term-II]*

Class	Frequency
0-15	10
15-30	7
30-45	x
45-60	15
60-75	10
75-90	12

Q 6. Find the mode of the following frequency distribution: *[CBSE 2022 Term-II]*

Class	Frequency
10-20	15
20-30	10
30-40	12
40-50	17
50-60	4

Short Answer Type-II Questions

Q 1. The arithmetic mean of the following frequency distribution is 53. Find the value of k. *[CBSE 2019]*

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	k	13

Q 2. Find the mean and the median of the marks of 100 students of a class, given in the following table: *[CBSE 2023]*

Marks	Number of students
0-5	4
5-10	11
10-15	13
15-20	15
20-25	31
25-30	26

Q 3. The mean of the following data is 42. Find the missing frequencies x and y, if the sum of frequencies is 100. *[CBSE 2015]*

Class interval	Frequency
0-10	7
10-20	10
20-30	x
30-40	13
40-50	y
50-60	10
60-70	14
70-80	9

Q 4. The length of 40 leaves of a plant are measured correct to nearest millimetre and the data obtained is represented in the following table: *[CBSE SQP 2023-24]*

Length (in mm)	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Find the average length of the leaves.

Q 5. Given below is the frequency distribution of the heights of players in a school.

Height (in cm)	Number of students
160-162	15
163-165	118
166-168	142
169-171	127
172-174	18

Find the modal height and interpret it.

Q 6. Following is the distribution of the long jump competition in which 250 students participated. Find the median distance jumped by the students. Interpret the median: *[CBSE SQP 2022, Term-II]*

Distance (in m)	Number of students
0-1	40
1-2	80
2-3	62
3-4	38
4-5	30

Long Answer Type Questions

Q 1. The following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life time of a lamp.

[CBSE SQP 2023-24]

Q 2. 250 apples of a box were weighed and the distribution of masses of the apples is given in the following table: *[CBSE 2023]*

Mass (in grams)	Number of apples
80-100	20
100-120	60
120-140	70
140-160	x
160-180	60

(i) Find the value of x and the mean mass of the apples.

(ii) Find the modal mass of the apples.

Q 3. The table given below shows the daily expenditure on food of 25 households in a locality:

Daily expenditure (₹)	Number of household
100-150	4
150-200	5
200-250	12
250-300	2
300-350	2

Find the mean daily expenditure on food. Also, find the mode of the data. *[CBSE 2023]*

Q 4. Mode of the following frequency distribution is 65 and sum of all the frequencies is 70. Find the missing frequencies x and y. *[CBSE 2015]*

Class	Frequency
0-20	8
20-40	11
40-60	x
60-80	12
80-100	y
100-120	9
120-140	9
140-160	5

Q 5. The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also, find the mode. *[CBSE SQP 2023-24]*

Marks obtained	Number of students
20-30	p
30-40	15
40-50	25
50-60	20
60-70	q
70-80	8
80-90	10

Q 6. Find the mean and the median of the following data: *[CBSE 2023]*

Marks	Number of Students
0-10	3
10-20	5
20-30	16
30-40	12
40-50	13
50-60	20
60-70	6
70-80	5

Q 7. Find the missing frequencies and the median for the following distribution, if the mean is 1.46.

Number of accidents	Number of days (frequency)
0	46
1	?
2	?
3	25
4	10
5	5
Total	200

Q 8. The monthly expenditure on milk in 200 families of a housing society is given below: *[CBSE 2023]*

Monthly expenditure (in ₹)	Frequency (periods)
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	x
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Find the value of x and also, find the median and mean expenditure on milk.

Solutions

Very Short Answer Type Questions

1. Class mark of 10-25 is $\frac{10+25}{2} = \frac{35}{2} = 17.5$

TR!CK

Class mark of an interval $= \frac{\text{Lower limit} + \text{Upper limit}}{2}$

and class mark of 35-55 is $\frac{35+55}{2} = \frac{90}{2} = 45$

2. $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 3 = \frac{3p+36}{15}$

$\Rightarrow 3 \times 15 = 3p + 36$

$\Rightarrow 45 - 36 = 3p$

$\Rightarrow 3p = 9$

$\Rightarrow p = \frac{9}{3} = 3$

Hence, the value of p is 3.

3.

x_i	f_i	$x_i f_i$
1.5	4	6
2	5	10
2.4	8	19.2
3	5	15
3.2	2	6.4
3.4	1	3.4
Total	25	60

Here, $\Sigma f_i = 25$ and $\Sigma x_i f_i = 60$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{60}{25} = 2.4$$

Hence, mean of gain in weight is 2.4 kg.

4. Given, mode = 8

Here, the terms 8 and 12 are repeated same number of times.

Hence, the value of x would be 8, since mode is the term which has the highest frequency.

5. In the given data, the highest frequency is 19. Therefore, its modal class is 80-90.

TiP

The succeeding value of modal class is just next to the modal class.

Hence, the succeeding value of modal class is 5.

6. On arranging the data in ascending order, we get 3, 6, 9, 11, 12, 19.

Here, $n = 6$ (even)

So, $\text{Median} = \frac{1}{2}\left\{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right\}$

$= \frac{1}{2}\left\{\left(\frac{6}{2}\right)\text{th term} + \left(\frac{6}{2}+1\right)\text{th term}\right\}$

$= \frac{1}{2}\{3\text{rd term} + 4\text{th term}\}$

$= \frac{1}{2}(9+11) = \frac{1}{2} \times 20 = 10$

7. The cumulative frequency distribution of more than type can be obtained as follows:

Production yield	Number of farms
More than 65	24
More than 60	24 + 30 = 54
More than 55	54 + 20 = 74
More than 50	74 + 16 = 90
More than 45	90 + 6 = 96
More than 40	96 + 4 = 100

8. The cumulative frequency table is shown below:

Height (in cm)	Number of students (f)	Cumulative frequency (cf)
140-150	5	5
150-160	6	11
160-170	10	21
170-180	4	25
180-190	7	32

Here, $\frac{N}{2} = \frac{32}{2} = 16$

Since, 16 lies in cumulative frequency 21. Therefore median class is 160-170. Now, the required sum of frequencies after median class is 4 + 7 *i.e.*, 11.

9. From Empirical relation,

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

$$12.4 = 3 \times \text{Median} - 2 \times 10.5$$

$$\Rightarrow \quad 12.4 = 3 \times \text{Median} - 21$$

$$\therefore \quad 3 \times \text{Median} = 12.4 + 21 = 33.4$$

$$\therefore \quad \text{Median} = \frac{33.4}{3} = 11.13$$

Short Answer Type-I Questions

1. We know that,

$$\text{Class mark } x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

The values of x_i and $f_i x_i$ can be calculated as follows:

CI	f_i	x_i	$f_i x_i$
0–2	1	1	$1 \times 1 = 1$
2–4	2	3	$2 \times 3 = 6$
4–6	1	5	$1 \times 5 = 5$
6–8	5	7	$5 \times 7 = 35$
8–10	6	9	$6 \times 9 = 54$
10–12	2	11	$2 \times 11 = 22$
12–14	3	13	$3 \times 13 = 39$
Total	$\sum f_i = 20$		$\sum f_i x_i = 162$

By direct method,

$$\text{Mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{162}{20} = 8.1$$

Hence, the mean number of plants per house is 8.1.

2. Given, mean = 12

The values of $f_i x_i$ can be calculated as follows:

x_i	f_i	$f_i x_i$
4	5	20
8	3	24
p	12	$12p$
16	5	80
20	4	80
Total	$\Sigma f_i = 29$	$\Sigma f_i x_i = 204 + 12p$

By direct method,

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 12 = \frac{204 + 12p}{29}$$

$$\Rightarrow \quad 12 \times 29 = 204 + 12p$$

$$\Rightarrow \quad 348 = 204 + 12p$$

$$\Rightarrow \quad 12p = 348 - 204 = 144$$

$$\Rightarrow \quad p = \frac{144}{12} = 12$$

Hence, the value of p is 12.

3.

Class interval	Mid value (x_i)	Frequency (f_i)	$d_i = x_i - A$, $A = 55$	$f_i d_i$
20-30	25	10	–30	–300
30-40	35	6	–20	–120
40-50	45	8	–10	–80
50-60	55	12	0	0
60-70	65	5	10	50
70-80	75	9	20	180
Total		50		–270

Here, $A = 55$, $h = 10$, $\Sigma f_i = 50$ and $\Sigma f_i d_i = -270$.

By using assumed mean method,

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 55 + \frac{(-270)}{50}$$

$$= 55 - 5.4 = 49.6$$

Hence, mean of given distribution table is 49.6.

4. Cumulative frequency distribution (of the less than type) table for the given data is as follows:

Less than type	cf
Less than 45	4
Less than 50	4 + 4 = 8
Less than 55	8 + 13 = 21
Less than 60	21 + 5 = 26
Less than 65	26 + 6 = 32
Less than 70	32 + 5 = 37
Less than 75	37 + 2 = 39
Less than 80	39 + 1 = 40

5. Given mode of distribution is 55, which lies in the interval 45-60.

So, 45-60 is a modal class.

$\therefore l = 45$, $f_m = 15$, $f_p = x$, $f_s = 10$ and $h = 15$

$$\because \quad \text{Mode} = l + \frac{f_m - f_p}{2f_m - f_p - f_s} \times h$$

$$55 = 45 + \frac{15 - x}{2 \times 15 - x - 10} \times 15$$

$$\Rightarrow \quad 10 = \frac{(15 - x)}{30 - x - 10} \times 15$$

$$\Rightarrow \quad 2(20 - x) = (15 - x) \times 3$$

$$\Rightarrow \quad 40 - 2x = 45 - 3x$$

$$\Rightarrow \quad x = 5$$

6. In given frequency distribution, the highest frequency is 17, which lies in the interval 40-50.

Thus, 40-50 is a modal class.

$\therefore l = 40$, $f_m = 17$, $f_p = 12$, $f_s = 4$ and $h = 10$

$$\because \quad \text{Mode} = l + \frac{f_m - f_p}{2f_m - f_p - f_s} \times h$$

$$\text{Mode} = 40 + \frac{17 - 12}{2 \times 17 - 12 - 4} \times 10$$

$$= 40 + \frac{5 \times 10}{34 - 16} = 40 + \frac{50}{18}$$

$$= 40 + 2.78 = 42.78$$

Short Answer Type-II Questions

1. The values of x_i and f_ix_i can be calculated as follows:

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	k	70 k
80-100	90	13	1170
Total		$\Sigma f_i = 72 + k$	$\Sigma f_i x_i =$ $3340 + 70\,k$

Here, $\Sigma f_i = 72 + k$ and $\Sigma f_ix_i = 3340 + 70k$

$\therefore$ $\text{Mean}\,(\overline{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 53 = \frac{3340 + 70k}{72 + k}$ (given)

$\Rightarrow$ $3816 + 53k = 3340 + 70k$

$\Rightarrow$ $17k = 476$

$\Rightarrow$ $k = 28$

Hence, the value of k is 28.

2. Calculation table for Mean and Median

Marks	Mid values (x_i)	Number of students (friquency) (f_i)	Cumulative frequency (cf)	$u_i = \frac{x_i - A}{h}$, $A = 17.5$, $h = 5$.	f_iu_i
0-5	2.5	4	4	-3	-12
5-10	7.5	11	15	-2	-22
10-15	12.5	13	28	-1	-13
15-20	17.5	15	43 = cf	0	0
20-25	22.5	31 = f	74	1	31
25-30	27.5	26	100	2	52
Total		$\Sigma f_i = 100 = N$			$\Sigma f_i u_i =$ 36

Here, $A = 17.5$, $h = 5$, $\Sigma f_i = 100$ and $\Sigma f_iu_i = 36$

By step-deviation method,

$\therefore$ Mean $= A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 17.5 + \frac{36}{100} \times 5$

$= 17.5 + \frac{36}{20} = 17.5 + 1.8 = 19.3$

Here, $\frac{N}{2} = \frac{100}{2} = 50$, which lies under the cumulative frequency 74. So, median class is (20-25).

$l = 20$, $f = 31$, $cf = 43$ and $h = 5$.

$\therefore$ Median $= l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h = 20 + \frac{(50 - 43)}{31} \times 5$

$= 20 + \frac{7 \times 5}{31} = 20 + \frac{35}{31} = 20 + 1.13 = 21.13$

Hence, mean and median of the marks of 100 students of a class are 19.3 and 21.13 respectively.

3. Given, mean = 42 and sum of frequencies = 100

$\therefore$ $7 + 10 + x + 13 + y + 10 + 14 + 9 = 100$

$\Rightarrow$ $63 + x + y = 100 \Rightarrow y = 100 - 63 - x$

$\Rightarrow$ $y = 37 - x$

The values of x_i and f_ix_i can be calculated as follows:

CI	f_i	x_i	f_ix_i
0-10	7	5	35
10-20	10	15	150
20-30	x	25	25x
30-40	13	35	455
40-50	$y=(37-x)$	45	$1665 - 45x$
50-60	10	55	550
60-70	14	65	910
70-80	9	75	675
Total	$\Sigma f_i = 100$		$\Sigma f_i x_i = 4440 - 20x$

By direct method,

$\therefore$ $\text{Mean}\ \overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow$ $42 = \frac{4440 - 20x}{100}$ (given)

$\Rightarrow$ $4200 = 4440 - 20x$

$\Rightarrow$ $20x = 4440 - 4200$

$\Rightarrow$ $x = \frac{240}{20} = 12$

$\therefore$ $y = 37 - 12 = 25$

Hence, $x = 12$ and $y = 25$.

4. Given data is not continuous, so we make continuous by subtracting and adding 0.5 in lower and upper limits respectively.

The continuous data is shown below:

Length (in mm)	Mid value (x_i)	No. of leaves (f_i)	$u_i = \frac{x_i - A}{h}$, $h = 9$, $A = 149$	f_iu_i
117.5-126.5	122	3	-3	-9
126.5 -135.5	131	5	-2	-10
135.5-144.5	140	9	-1	-9
144.5-153.5	149	12	0	0
153.5-162.5	158	5	1	5
162.5-171.5	167	4	2	8
171.5-180.5	176	2	3	6
Total		$\Sigma f_i = 40$		$\Sigma f_iu_i = -9$

Here, $\Sigma f_i = 40$, $\Sigma f_iu_i = -9$, $A = 149$ and $h = 9$.

By step-deviation method,

$\text{Mean}\ (\overline{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 149 + \frac{(-9)}{40} \times 9$

$= 149 - \frac{81}{40} = 149 - 2.025 = 146.975$

So, average length of the leaves is 146.957 mm.

5. The given frequency distribution is not continuous, so first we convert it into continuous frequency distribution.

TR!CK

To convert the given distribution into continuous distribution, we subtract 0.5 *in lower limit and add* 0.5 *in upper limit of each interval.*

Class interval	Frequency
159.5-162.5	15
162.5-165.5	118
165.5-168.5	142
168.5-171.5	127
171.5-174.5	18

∵ Class interval 165.5-168.5 has maximum frequency, so it is the modal class.

Here, $l = 165.5$, $f_m = 142$, $f_p = 118$, $f_s = 127$ and $h = 3$.

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_p}{2f_m - f_p - f_s}\right) \times h$$

$$= 165.5 + \left(\frac{142 - 118}{2 \times 142 - 118 - 127}\right) \times 3$$

$$= 165.5 + \frac{24}{39} \times 3$$

$$= 165.5 + 1.85 = 167.35$$

Hence, the modal height is 167.35 cm. This means that the height of maximum number of players in the school is 167.35 cm (approx).

6. The cumulative frequency of given distribution can be calculated as follows:

Distance (in m)	Number of students (f_i)	Cumulative frequency (cf)
0-1	40	40
1-2	80	120
2-3	62	182
3-4	38	220
4-5	30	250

Here, $\frac{N}{2} = \frac{250}{2} = 125$

∵ 125 lies under Cf 182, so median class is 2-3.

$\therefore$ $l = 2$, $f = 62$, $Cf = 120$ and $h = 1$

$$\text{Median} = l + \frac{\frac{N}{2} - Cf}{f} \times h$$

$$= 2 + \frac{125 - 120}{62} \times 1$$

$$= 2 + \frac{5}{62}$$

$$= 2 + 0.08 = 2.08 \text{ m}$$

Hence, the median distance is 2.08 m. This means that 50% students jump below 2.08 m and 50% above it.

Long Answer Type Questions

1. Calculation table for Mean (average):

Life time (in hours)	Mid values (x_i)	Number of lamps (frequency) (f_i)	$u_i = \frac{x_i - A}{h}$, $A = 3250$, $h = 500$	f_iu_i
1500-2000	1750	14	–3	–42
2000-2500	2250	56	–2	–112
2500-3000	2750	60	–1	–60
3000-3500	3250	86	0	0
3500-4000	3750	74	1	74
4000-4500	4250	62	2	124
4500-5000	4750	48	3	144
Total		$\Sigma f_i = 400$		$\Sigma f_iu_i = 128$

Here, $A = 3250$, $\Sigma f_i = 400$, $\Sigma f_iu_i = 128$ and $h = 500$

By step-deviation method,

$$\therefore \text{ Mean } (\bar{x}) = A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$= 3250 + \frac{128}{400} \times 500$$

$$= 3250 + 32 \times 5$$

$$= 3250 + 160 = 3410$$

So, average life time of a lamp is 3410 hours.

2. (i) Given, number of apples in a box = 250.

$\therefore$ $20 + 60 + 70 + x + 60 = 250$

$\Rightarrow$ $210 + x = 250$

$\Rightarrow$ $x = 250 - 210 = 40$.

Calculation table for mean

Mass (in grams)	Mid values (x_i)	No. of apples (f_i)	$u_i = \frac{x_i - A}{h}$, $A = 130$, $h = 20$	f_iu_i
80-100	90	20	–2	–40
100-120	110	60	–1	–60
120-140	130	70	0	0
140-160	150	$x = 40$	1	40
160-180	170	60	2	120
Total		$\Sigma f_i = 250$		$\Sigma f_iu_i = 60$

Here, $\Sigma f_i = 250$, $\Sigma f_iu_i = 60$, $h = 20$ and $A = 130$

Using step deviation method,

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h = 130 + \frac{60}{250} \times 20$$

$$\Rightarrow \quad = 130 + \frac{1200}{250} = 130 + 4.8 = 134.8$$

So, mean mass of the apples is 134.8 grams.

(ii) From the table, (120-140) is a modal class, because it has a maximum frequency, *i.e.*, 70.

$\therefore\ l = 120,\ f_m = 70,\ f_p = 60,\ f_s = 40$ and $h = 20$

$$\text{Mode} = l + \frac{f_m - f_p}{2f_m - f_p - f_s} \times h = 120 \times \frac{70-60}{2\times 70 - 60 - 40} \times 20$$

$$= 120 + \frac{10\times 20}{140-100} = 120 + \frac{200}{40} = 120 + 5$$

$$= 125$$

So, the modal mass of the apples is 125 gm.

3. The values of x_i and f_ix_i can be calculated as follows:

Daily expendi -ture in (₹)	**Frequency (f_i)**	**Class marks (x_i)**	$f_i x_i$
100-150	4	125	500
150-200	5	175	875
200-250	12	225	2700
250-300	2	275	550
300-350	2	325	650
Total	$\Sigma f_i = 25$		$\Sigma f_i x_i =$ 5275

$\therefore$ The mean of daily expenditure of food.

$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{5275}{25} = 211$$

Hence, mean of daily expenditure food is ₹ 211

The highest frequency in the given table is 12, whose corresponding interval is 200-250. So modal class is 200-250.

Here, $l = 200,\ f_m = 12,\ f_p = 5,\ f_s = 2$ and $h = 50$

$$\therefore \text{ Mode } = l + \frac{f_m - f_p}{2f_m - f_p - f_s} \times h$$

$$= 200 + \frac{12-5}{2\times 12 - 5 - 2} \times 50 = 200 + \frac{350}{17}$$

$$= 200 + 20.59 = ₹\ 220.59$$

Hence, mode of daily expenditure of food is ₹220.59.

4. Given, mode = 65

$\therefore$ Modal class is 60-80.

Class interval	**f**
0-20	8
20-40	11
40-60	$x \to f_p$
60-80	$12 \to f_m$
80-100	$y \to f_s$
100-120	9
120-140	9
140-160	5

Also, sum of all the frequencies = 70

$\Rightarrow$ $54 + x + y = 70$

$\Rightarrow$ $x + y = 70 - 54$

$\Rightarrow$ $x + y = 16$ ...(1)

Here, $l = 60,\ f_m = 12,\ f_p = x,\ f_s = y$ and $h = 20$.

$$\text{Mode} = l + \left(\frac{f_m - f_p}{2f_m - f_p - f_s}\right) \times h$$

$\Rightarrow$ $$65 = 60 + \left(\frac{12-x}{2\times 12 - x - y}\right) \times 20$$

$\Rightarrow$ $$65 - 60 = \left(\frac{12-x}{24 - x - y}\right) \times 20$$

$\Rightarrow$ $5(24 - x - y) = 20(12 - x)$

$\Rightarrow$ $24 - x - y = 4(12 - x)$

$\Rightarrow$ $24 - x - y = 48 - 4x$

$\Rightarrow$ $3x - y = 24$ ...(2)

On adding eqs. (1) and (2), we get.

$4x = 40 \Rightarrow x = 10$

$\therefore$ $10 + y = 16$ [from eq. (1)]

$\Rightarrow$ $y = 16 - 10 = 6$

Hence, $x = 10$ and $y = 6$.

5. Given, the sum of all frequencies = 90

$\therefore$ $p + 15 + 25 + 20 + q + 8 + 10 = 90$

$\Rightarrow$ $p + q = 90 - 78 - 12$

$\Rightarrow$ $q = 12 - p$ ..... (1)

Calculation table for median

Marks	**Number of students**	**Cumulative frequency (cf)**
20-30	p	p
30-40	15	$p + 15$
40-50	25	$p + 40$
50-60	20	$p + 60$
60-70	q	$p + q + 60$
70-80	8	$p + q + 68$
80-90	10	$p + q + 78$

Given, the median of the above data is 50.

50 lies in the interval 50-60. So, median class is (50-60).

Here, $l = 50,\ \frac{N}{2} = \frac{p+q+78}{2},\ f = 20,\ cf = p + 40$ and $h = 10$.

$\therefore$ $$\text{Median } = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times f$$

$\Rightarrow$ $$50 = 50 + \frac{\left\{\left(\frac{p+q+78}{2}\right) - (p+40)\right\}}{20} \times 10$$

$\Rightarrow$ $p + q + 78 - 2p - 80 = 0$ [from eq. (1)]

$\Rightarrow$ $p - q = -2 \Rightarrow p - 12 + p = -2$

$\Rightarrow$ $2p = 10 \Rightarrow p = 5$

$\Rightarrow$ $q = 12 - 5 = 7$

From the table, Interval (40-50) is a modal class, because it has a maximum frequency.

Here, $l = 40,\ f_m = 25,\ f_p = 15,\ f_s = 20$ and $h = 10$.

$\therefore$ $$\text{Mode} = l + \frac{f_m - f_p}{2\times f_m - f_p - f_s} \times h$$

$$= 40 + \frac{25-15}{2\times 25-15-20}\times 10 = 40 + \frac{100}{50-35}$$

$$= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$$

6. Calculation table for Mean and Median

Number of cars	Mid values (x_i)	Frequency (f_i)	Cumulative frequency cf	$u_i = \frac{x_i - A}{h}$, $A = 45$, $h = 10$	$f_i u_i$
0-10	5	3	3	–4	–12
10-20	15	5	8	–3	–15
20-30	25	16	24	–2	–32
30-40	35	12	36	–1	–12
40-50	45	13	49	0	0
50-60	55	20	69	1	20
60-70	65	6	75	2	12
70-80	75	5	80	3	15
Total		$\Sigma f_i = 80 = N$			$\Sigma f_i u_i = -22$

Here, $A = 45$, $h = 10$, $\Sigma f_i = 80$ and $\Sigma f_i u_i = -22$.
By step-deviation method.

$$\therefore \quad \text{Mean}\,(\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i}\times h = 45 + \frac{(-22)}{80}\times 10$$

$$= 45 - 2.75 = 42.25$$

Here $\frac{N}{2} = \frac{80}{2} = 40$ which lies under the cumulative frequency 49 so, median class is (40-50).
$l = 40$, $f = 13$, $cf = 36$ and $h = 10$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f}\times h = 40 + \frac{(40-36)}{13}\times 10$$

$$= 40 + \frac{4}{13} = 40 + 3.08 = 43.08$$

8. Given , number of families = 200

$24 + 40 + 33 + x + 30 + 22 + 16 + 7 = 200$

$\Rightarrow \quad 172 + x = 200$

$\Rightarrow \quad x = 200 - 172 = 28$

Calculation table for mean and medium

Monthly expenditure (in ₹)	Mid value (x_i)	No. of families (f_i)	Cumulative frequency (cf)	$u_i = \frac{x_i - A}{h}$, $A = 3250$, $h = 500$	$f_i u_i$
1000-1500	1250	24	24	–4	– 96
1500 -2000	1750	40	64	–3	– 120
2000-2500	2250	33	97	–2	– 66
2500.-3000	2750	x = 28	125	–1	–28
3000-3500	3250	30	155	0	0
3500-4000	3750	22	177	1	22
4000-4500	4250	16	193	2	32
4500-5000	4750	7	200	3	21
Total		$\Sigma f_i = 200$			$\Sigma f_i u_i = -235$

7. Let the missing frequencies corresponding to 1 and 2 are f_1 and f_2 respectively. Then

Sum of frequencies $= 200 = \Sigma f = N$

$\Rightarrow \quad 46 + f_1 + f_2 + 25 + 10 + 5 = 200$

$\Rightarrow \quad f_1 + f_2 = 200 - 86 = 114 \quad \ldots(1)$

x_i	0	1	2	3	4	5
f_i	46	f_1	f_2	25	10	5
$f_i x_i$	0	f_1	$2f_2$	75	40	25

By direct method.

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = 1.46 \quad \text{(given)}$$

$$\Rightarrow \quad \frac{0 + f_1 + 2f_2 + 75 + 40 + 25}{200} = 1.46$$

$$\Rightarrow \quad f_1 + 2f_2 + 140 = 1.46 \times 200$$

$$\Rightarrow \quad f_1 + 2f_2 = 292 - 140 = 152 \quad \ldots(2)$$

On solving eqs. (1) and (2). we get

$f_2 = 38$ and $f_1 = 76$

The new formed table is shown below.

x	0	1	2	3	4	5
f	46	76	38	25	10	5
cf	46	122	160	185	195	200

Here, $N = 200$. $\frac{N}{2} = \frac{200}{2} = 100$. even

$$\text{Median} = \frac{\left(\frac{N}{2}\right)\text{th term} + \left(\frac{N}{2}+1\right)\text{th term}}{2}$$

$$= \frac{100\text{th term} + 101\text{th term}}{2}$$

$$= \frac{1+1}{2} = \frac{2}{2} = 1$$

(Since, both 100th and 101th terms lie in cf 122, so, the corresponding value of x is 1.)

Hence, the missing frequencies are 76 and 38 and the required median is 1.

Here, $N = 200 \Rightarrow \frac{N}{2} = \frac{200}{2} = 100$ which lies under the cumulative frequency 125. So, median class is (2500-3000).

$l = 2500$, $cf = 97$, $f = 28$ and $h = 500$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h = 2500 + \frac{(100-97)}{28} \times 500$$

$$= 2500 + \frac{1500}{28} = 2500 + 53.57 = 2553.57$$

Also, by step-deviation method,

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 3250 + \frac{(-235)}{200} \times 500$$

$$= 3250\text{-}587.5$$

$$= 2662.5$$

Hence, median and mean expenditure on milk are ₹ 2553.57 and ₹ 2662.5 respectively.

Chapter Test

Multiple Choice Questions

Q 1. The mean and median of a distribution are 12 and 14, respectively. The value of the mode is:

a. 16 b. 18 c. 20 d. 24

Q 2. The sum of the class marks of the classes 20-30 and 50-60 is:

a. 80 b. 70 c. 60 d. 50

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): The median of the following frequency distribution is 40:

Variate	25	31	34	40	45	48	50	60
Frequency	3	8	10	15	10	9	6	2

Reason (R): If we increase the frequency of initial and last variate, then their median remain same.

Q 4. Assertion (A): The modal class of the following data is 55-60:

Class intervals	Frequency
50-55	5
55-60	20
60-65	10
65-70	10
70-75	9
75-80	6
80-85	12
85-90	8

Reason (R): The preceding value of modal class is 10.

Fill in the Blanks

Q 5. If each observation is adding by 3, then new mean will be increased by .....

Q 6. The sum of the limits of median class in the following frequency distribution table is .....

Number of plants	0-5	5-10	10-15	15-20	20-25	25-30
Number of houses	3	4	6	4	8	5

True/False

Q 7. In the frequency distribution, if $\Sigma f_i x_i = 800$ and $\Sigma f_i = 40$, the mean of the distribution is 40.

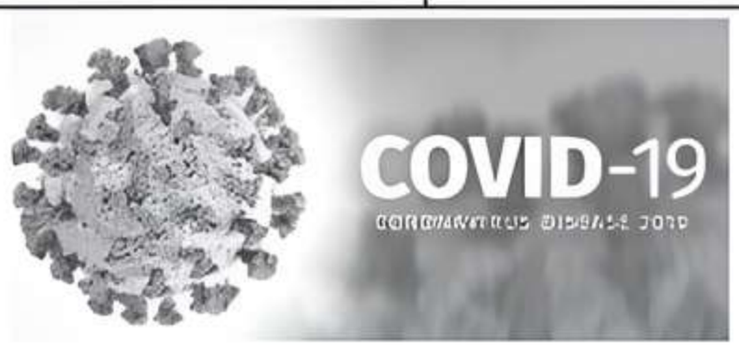
Q 8. In the continuous distribution data, mode is always lies in the interval in which the highest frequency exist.

Case Study Based Question

Q 9. Household income in India was drastically impacted due to the COVID-19 lockdown. Most of the companies decided to bring down the salaries of the employees by 50%.

The following table shows the salaries (in percent) received by 25 employees during lockdown:

Salaries received (in percent)	Number of employees
50-60	9
60-70	6
70-80	8
80-90	2



Based on the given information, solve the following questions:

(i) Find the modal class of the given data.

(ii) Find the median class of the given data.

(iii) Find the mean of the given data.

OR

Find the mode of the given data.

Very Short Answer Type Questions

Q 10. If the mode of the data 3, 5, 8, 9, 8, 12, 7, 12 and x is 8, find the value of x.

Q 11. Find the median of 3, 9, 6, 12, 19 and 11.

Short Answer Type-I Questions

Q 12. If the mean of the following data is 12, then find the value of p:

x_i	4	8	p	16	20
f_i	5	3	10	5	4

Q 13. The following is the distribution of weights (in kg) of 40 persons:

Weight (in kg)	Number of persons
40-45	4
45-50	4
50-55	13
55-60	5
60-65	6
65-70	5
70-75	2
75-80	1

Construct a cumulative frequency distribution (of the less than type) table for the above data.

Short Answer Type-II Questions

Q 14. The daily wages of 80 workers in a project are given below:

Wages (in ₹)	Number of workers
400-450	2
450-500	6
500-550	12
550-600	18
600-650	24
650-700	13
700-750	5

Find the median daily wages of the workers.

Q 15. A survey conducted on 20 families in a locality by a group of students resulted in the following frequency table for the number of family members in a family. *[CBSE 2023]*

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Determine the mean and mode of the above data.

Long Answer Type Question

Q 16. Find the mean and the median of the following data: *[CBSE 2023]*

Class	Frequency
85-90	10
90-95	12
95-100	15
100-105	14
105-110	12
110-115	7